Journal of Applied Mechanics and Technical Physics, Vol. 42, No. 3, pp. 538–545, 2001

LARGE IRREVERSIBLE LOCAL SLIPS OF A LOOSE FREE-FLOWING MATERIAL UNDER HARD SIGN-VARIABLE QUASISTATIONARY LOADING

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UDC 539.215

The theory of slip for the case of large strains is considered. Irreversible distortion by slippage is represented as a Truesdell transformation of simple shear and is specified with respect to the unstrained configuration. It is proved that a small local slip leads decreases tangential stresses on the slip area and changes the strained stress over the entire region. The calculated orientations of the local slip systems are compared to results of classical experiments on sign-variable quasistationary deformation of loose free-flowing materials.

The classical theory of slip was proposed by Batdorf and Budyanskii [1] for the case of small strains. Asaro and Rice [2] and Asaro [3] extended it to the case of large strains. In this case too, allowance for the mutual effect of local slips remains a main problem (see [4]). To solve this problem, Leonov an Rychkov [5] proposed to sum up slips on all slip planes and in all directions in each of the planes and to define the relationship between the stress tensor and small slips by a deformation law. Rusinko [6] proposed a dependence of the slipping friction in each slip system on the action of other such systems. Mokhel' et al. [7] introduced a function describing the mutual effect of slips.

In the present paper, it is proposed to describe the mutual effect of slips taking into account the finiteness of strains within the framework of the kinematic approach without invoking additional equations of state or parameters. In this case, as in experiments, slips acquire the properties of localization and self-organization of orientations. Results of numerical calculations are compared to experimental results [8] on quasistationary sign-variable loading of free-flowing media.

1. Kinematics, Stresses in Various Configurations, and Elasticity. We assume that in a small neighborhood deformed by strain gradient A_{ij} there is an interface given by orthogonal coordinates $\boldsymbol{e}_k^{(0)}$ $(k = \alpha, \beta, \gamma)$ with normal line $\boldsymbol{e}_\alpha^{(0)}$. Under the assumption of no slips over the interface, the strained state can be defined as for a continuous neighborhood:

$$A_{ij} = \frac{\partial R_i}{\partial r_j}. \tag{1}$$

The adopted relation (see [9]) defines distortions $C_{kj}^{(0)}$ and rotations B_{ik}:

$$A_{ij} = B_{ik} C_{kj}^{(0)}, \tag{2}$$

so that

$$C_{ij}^{(0)} C_{jk}^{(0)} = A_{ik} A_{jk}. \tag{3}$$

Here and below, the superscripts (0), (1), and (2) correspond to the state of the object in the unstrained configuration of the region, in the strained but unrotated configuration, and in the final (actual) configuration, respectively.

For a symmetric distortion tensor $C_{kj}^{(0)}$, one finds the principal orthogonal directions $\boldsymbol{e}_\alpha^{(0)}$ with elongation ratio $c_\alpha^{(0)}$ can be found:

Kuban' State University, Krasnodar 350040. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 42, No. 3, pp. 186–196, May–June, 2001. Original article submitted May 17, 1999; revision submitted September 11, 2000.

 0021-8944/01/4203-0538 $25.00

$$c_\alpha^{(0)} \boldsymbol{e}_\alpha^{(0)} = \boldsymbol{C}^{(0)} \boldsymbol{e}_\alpha^{(0)} \tag{4}$$

(no summation over Greek coefficients).

As the measure of general strains in the region, we use relative elongations. It should be noted that use of this measure does not lead to any restrictions on the values of the strains and rotations described:

$$\varepsilon_{ij}^{(0)} = C_{ij}^{(0)} - \delta_{ij}. \tag{5}$$

The stress–strain relation is specified for objects of the same configuration. For this, we introduce the so-called conditional stresses $\sigma_{\alpha\alpha}^{(0)}$ ($\alpha = 1$–3) related to the principal areas of the unstrained state. Because in the present paper, anisotropic effects are introduced exclusively via irreversible slips, the elastic relation is represented as the classical isotropic relations of Hooke's law. In this case, for the principal values, we have

$$\sigma_{\alpha\alpha}^{(0)} = \lambda \varepsilon_{kk}^{(0)} + 2G\varepsilon_{\alpha\alpha}^{(0)}. \tag{6}$$

Here λ and G are Lamé parameters.

The actual stresses related to the strained configuration in the orthogonal system of principal directions of deformation $\boldsymbol{e}_\alpha^{(1)} \equiv \boldsymbol{e}_\alpha^{(0)}$ are given by

$$\sigma_{\alpha\alpha}^{(1)} = \sigma_{\alpha\alpha}^{(0)}/(c_\beta^{(0)} c_\gamma^{(0)}) \qquad (\alpha \neq \beta, \quad \alpha \neq \gamma, \quad \beta \neq \gamma). \tag{7}$$

The actual stresses in the strained but unrotated configuration are defined by the relations

$$\sigma_{ij}^{(1)} = e_{ik}^{(1)} \sigma_{kl}^{(1)} e_{jl}^{(1)} \tag{8}$$

and, finally, in the actual configuration, by the relations

$$\sigma_{kl}^{(2)} = B_{ki}^{(1)} \sigma_{ij}^{(1)} B_{lj}^{(1)}. \tag{9}$$

2. Local Interface. Stresses, Slip Condition, and Description of Simple Slip. Let the region considered contain a certain interface. We assume that a quasiequilibrium state of the region being deformed can be achieved as a result of stresses occurring during deformation of the unstrained configuration of the region due to elastic interaction of the structural elements of the medium and reduction in the resulting overstresses by irreversible distortion of the slip structure.

We choose a new orthogonal coordinate system $\boldsymbol{e}_i^{(0)}$ ($i = \alpha, \beta, \gamma$) related to the orientation of the interface in the unstrained state, and direct the $\boldsymbol{e}_\alpha^{(0)}$ axis normally to the area considered. The values of the stress tensor components $\sigma_{ij}^{(1)}$ obtained from relations (7) for the distorted configuration can be used to determine the tangential (τ_α) and normal ($\sigma_{\alpha\alpha}$) stress components acting on the α-interface with allowance for its new position after distortion of the region. Indeed, the orthogonal tangential vectors $\boldsymbol{e}_\beta^{(0)}$ and $\boldsymbol{e}_\gamma^{(0)}$ are converted to the nonorthogonal vectors $\boldsymbol{e}_\beta^{(1)}$ and $\boldsymbol{e}_\gamma^{(1)}$, respectively, with change in the moduli:

$$\boldsymbol{e}_\beta^{(1)} = \boldsymbol{C}^{(0)} \boldsymbol{e}_\beta^{(0)}, \qquad \boldsymbol{e}_\gamma^{(1)} = \boldsymbol{C}^{(0)} \boldsymbol{e}_\gamma^{(0)}. \tag{10}$$

The change in the surface area of the α-interface S/S_0 is given by

$$S/S_0 = |\boldsymbol{e}_\beta^{(1)} \times \boldsymbol{e}_\gamma^{(1)}|. \tag{11}$$

The normal vector $\boldsymbol{e}_\alpha^{(1)}$ to the α-interface in the distorted configuration is given by the relation

$$\boldsymbol{e}_\alpha^{(1)} = \boldsymbol{e}_\beta^{(1)} \times \boldsymbol{e}_\gamma^{(1)}. \tag{12}$$

In what follows, it is convenient to use the normalization

$$\boldsymbol{e}_\alpha^{(1)} = \boldsymbol{e}_\alpha^{(1)}/|\boldsymbol{e}_\alpha^{(1)}|. \tag{13}$$

Then, the total stress vector components $\boldsymbol{\sigma}_\alpha^{(1)}$ acting on the α-interface and related to the distorted configuration are given by the expression

$$\sigma_{i(\alpha)}^{(1)} = \sigma_{ij}^{(1)} e_{j(\alpha)}^{(1)}. \tag{14}$$

The projection of the total stress vector onto the normal $\boldsymbol{e}_\alpha^{(1)}$ to the surface is

$$\sigma_{\alpha\alpha}^{(1)} = \sigma_{i(\alpha)}^{(1)} e_{i(\alpha)}^{(1)} = e_{(\alpha)i}^{(1)} \sigma_{ij}^{(1)} e_{j(\alpha)}^{(1)} \tag{15}$$

and the normal components of the total stress vector are

$$\sigma^{(1)}_{i(\alpha\alpha)} = \sigma^{(1)}_{\alpha\alpha} e^{(1)}_{i(\alpha)}; \tag{16}$$

then, the total tangential stress components over the α-interface are

$$\tau^{(1)}_{i(\alpha)} = \sigma^{(1)}_{i(\alpha)} - \sigma^{(1)}_{i(\alpha\alpha)}. \tag{17}$$

We note that the tangential component is in the plane of the vectors $\boldsymbol{e}^{(1)}_{\beta}$ and $\boldsymbol{e}^{(1)}_{\gamma}$.

The absolute value of the tangential component of the total stress acting on the α-interface is

$$\tau^{(1)}_{\alpha} = \sqrt{\tau^{(1)}_{i(\alpha)} \tau^{(1)}_{i(\alpha)}}. \tag{18}$$

The absolute values of the tangential stress component $\tau^{(1)}_{\alpha}$ and normal stress component $\sigma^{(1)}_{\alpha\alpha}$ stress components acting on the α-interface shows the possibility of slippage over the internal interface coinciding with the α-interface if the stresses are normalized to the surface area of the α-interface in the unstrained configuration:

$$\tau^{(0)}_{\alpha} = \tau^{(1)}_{\alpha} S_0/S, \qquad \sigma^{(0)}_{\alpha\alpha} = \sigma^{(1)}_{\alpha\alpha} S_0/S. \tag{19}$$

In this case, we assume that the possibility of slippage over these areas is determined only by the stresses acting on their surfaces.

Let us assume that if the stresses acting on the α-interface considered exceed a certain level $f_{\alpha}(\sigma^{(0)}_{\alpha\alpha}, \tau^{(0)}_{\alpha})$, slippage of this interface occurs in the region. The slippage condition is specified by the simplest relation for dry friction. In particular, for conditional stresses, we have

$$f_{\alpha}(\sigma^{(0)}_{\alpha\alpha}, \tau^{(0)}_{\alpha}) \equiv k\sigma^{(0)}_{\alpha\alpha} + \tau^{(0)}_{\alpha} \geqslant 0, \tag{20}$$

where k is the friction coefficient on the interface. With allowance for (19), the limiting relation (20) can also be written for actual stresses:

$$f_{\alpha}(\sigma^{(1)}_{\alpha\alpha}, \tau^{(1)}_{\alpha}) \equiv k\sigma^{(1)}_{\alpha\alpha} + \tau^{(1)}_{\alpha} \geqslant 0. \tag{21}$$

We assume that the direction of irreversible slip over the interface considered coincides with the direction of reversible shear deformation on it. In contrast to Schmidt tangential stresses [3], the irreversible slip transformation, as well as the reversible deformation components, refer to the same (unstrained) configuration of the region. Then, the reversible strain of the region relative to the unstrained configuration in the coordinate system $\boldsymbol{e}^{(0)}_{k}$ $(k = \alpha, \beta, \gamma)$ attached to the α-interface is given by

$$\varepsilon^{(0)}_{lk} = e^{(0)}_{il} \varepsilon^{(0)}_{ij} e^{(0)}_{jk}, \tag{22}$$

and the matrix of the strain tensor obtained has the form

$$\varepsilon^{(0)}_{lk} = \begin{pmatrix} \varepsilon^{(0)}_{\alpha\alpha} & \varepsilon^{(0)}_{\alpha\beta} & \varepsilon^{(0)}_{\alpha\gamma} \\ \varepsilon^{(0)}_{\beta\alpha} & \varepsilon^{(0)}_{\beta\beta} & \varepsilon^{(0)}_{\beta\gamma} \\ \varepsilon^{(0)}_{\gamma\alpha} & \varepsilon^{(0)}_{\gamma\beta} & \varepsilon^{(0)}_{\gamma\gamma} \end{pmatrix}. \tag{23}$$

The total shear strain over the α-interface is

$$\varepsilon^{(0)}_{\alpha} = \sqrt{\varepsilon^{(0)}_{i\alpha} \varepsilon^{(0)}_{i\alpha}} \qquad (i = \beta, \gamma). \tag{24}$$

The irreversible distortion by slip A^{sh}_{ij} over the α-interface is represented as a simple shear transformation [9, 10] with shear components proportional to the corresponding strain tensor components $\varepsilon^{(0)}_{ij}$:

$$A^{\text{sh}}_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ \varepsilon^{\text{sh}}_{\beta\alpha} & 1 & 0 \\ \varepsilon^{\text{sh}}_{\gamma\alpha} & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ \chi\varepsilon^{(0)}_{\beta\alpha} & 1 & 0 \\ \chi\varepsilon^{(0)}_{\gamma\alpha} & 0 & 1 \end{pmatrix}, \tag{25}$$

which distorts the unstrained configuration. Here χ is a small positive slip parameter. The introduced transformation does not change the volume of the region considered.

The total irreversible shear $\varepsilon^{\text{sh}}_{\alpha}$ over the α-interface is

$$\varepsilon^{\text{sh}}_{\alpha} = \sqrt{\varepsilon^{\text{sh}}_{i\alpha} \varepsilon^{\text{sh}}_{i\alpha}} \qquad (i = \beta, \gamma). \tag{26}$$

The shear parameter χ is represented as a positive function of the excess of shear stresses over restraining stresses (18):

$$\chi = f_\alpha(\sigma^{(0)}_{\alpha\alpha}, \tau^{(0)}_\alpha)\Delta t/\nu^{\rm sh} \equiv (k\sigma^{(0)}_{\alpha\alpha} + \tau^{(0)}_\alpha)\Delta t/\nu^{\rm sh}. \tag{27}$$

Here $\nu^{\rm sh}$ is the shear viscosity on the local slip area and Δt is the step in time. The obtained irreversible shear transformation $A^{\rm sh}_{kj}$ is "subtracted" from the strain gradient matrix:

$$A_{ij} = A^0_{lk} A^{\rm sh}_{kj}. \tag{28}$$

Here A^0_{kl} is the strain gradient relative to the new unstrained configuration obtained after the irreversible shear $A^{\rm sh}_{kj}$. Relation (28) leads to the relation

$$\boldsymbol{A}^0 = \boldsymbol{A}(\boldsymbol{A}^{\rm sh})^{-1}. \tag{29}$$

In turn, the strain gradient relative to the new unstrained configuration A^0_{kl} can be expanded polarly:

$$A^0_{ij} = B_{ik} C^{(0)}_{kl}. \tag{30}$$

Here $C^{(0)}_{kl}$ is the reversible (in this case) distortion of the region relative to the new unstrained configuration and B_{ik} is the rotation of the distorted configuration into the actual configuration.

The structure of relations (28)–(30) shows that the irreversible shear changes the strained stress of the subregion, and the irreversible shears defined by relation (25) decreases the tangential stresses on the α-slip area [10]. The last statement can be formulated as the following theorem.

Theorem 1. *In a deformed elastic body, a small slip over a local area in the direction of general shear deformations leads to a decrease in tangential stresses over the area considered.*

We consider a strained region at the moment when shear on the α-area has not yet occurred, in the case of small strains. Then, the strain gradient expansion (1) reduces to representation of the displacement gradient in the form of the sum of tensor of small strains ε_{ij} and small rotations ω_{ij}. In this case, in a coordinate system attached to the α-area considered, $u_{i,j} = \omega_{ij} + \varepsilon_{ij}$, where $\omega_{ij} = (u_{i,j} - u_{j,i})/2$ and $\varepsilon_{ij} = (u_{i,j} + u_{j,i})/2$. It should be noted that since slippage has not yet occurred in the region, the strains ε_{ij} are elastic and reversible.

To describe slippage over the α-area, we write the simple shear displacements gradient $u^{\rm sh}_{ij}$ for infinitesimal strains similarly to the slip transformation for large strains

$$u^{\rm sh}_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ \varepsilon_{\beta\alpha}\, d\chi & 0 & 0 \\ \varepsilon_{\gamma\alpha}\, d\chi & 0 & 0 \end{pmatrix}$$

and consider the change in the stress–strained state of the region after such slip. For this, in the displacement gradient $u_{i,j}$, we choose the irreversible slip $u^{\rm sh}_{ij}$ and obtain the displacement gradient $u^0_{i,j}$ relative to the new unstrained state $u_{i,j} = u^0_{i,j} + u^{\rm sh}_{ij}$. From this, for the displacement gradient components in relation to the α-area, we obtain $u^0_{\beta,\alpha} = u_{\beta,\alpha} - u^{\rm sh}_{\beta\alpha} = u_{\beta,\alpha} - \chi\varepsilon_{\beta\alpha}$ and $u^0_{\gamma,\alpha} = u_{\gamma,\alpha} - u^{\rm sh}_{\gamma\alpha} = u_{\gamma,\alpha} - \chi\varepsilon_{\gamma\alpha}$, where $u^0_{\alpha,\beta} = u_{\alpha,\beta}$, and $u^0_{\gamma,\beta} = u_{\gamma,\beta}$. Then, the reversible strain components after irreversible slip are defined by the relations $\varepsilon^0_{ij} = (u^0_{i,j} + u^0_{i,j})/2$. For the α-area, we have $\varepsilon^0_{\beta\alpha} = (u^0_{\beta,\alpha} + u^0_{\alpha,\beta})/2 = (u_{\beta,\alpha} + u_{\alpha,\beta} - \chi\varepsilon_{\beta\alpha})/2 = \varepsilon_{\beta\alpha} - \varepsilon_{\beta\alpha}\, d\chi/2 = \varepsilon_{\beta\alpha}(1 - d\chi/2)$. Similarly, $\varepsilon^0_{\gamma\alpha} = \varepsilon_{\gamma\alpha}(1 - d\chi/2)$. Because, by definition, $d\chi$ is a positive infinitesimal parameter, it follows that $1 - d\chi/2 < 1$, and, accordingly, after the irreversible slip, the reversible strains on the α-area decrease in absolute value: $|\varepsilon^0_{\beta\alpha}| < |\varepsilon_{\beta\alpha}|$ and $|\varepsilon^0_{\gamma\alpha}| < |\varepsilon_{\gamma\alpha}|$. The relations obtained imply a decrease in absolute values of the tangential stress components on the irreversible slip area. Indeed, let the elastic medium be characterized by, at least, elastic shear modulus G. Then, because the right and left sides of the last inequalities contain the elastic strain components, we obtain $|2G\varepsilon^0_{\beta\alpha}| < |2G\varepsilon_{\beta\alpha}|$ and $|2G\varepsilon^0_{\gamma\alpha}| < |2G\varepsilon_{\gamma\alpha}|$. Finally, for tangential stresses before and after the slip, we obtain, respectively, $|\tau^0_{\beta\alpha}| < |\tau_{\beta\alpha}|$ and $|\tau^0_{\gamma\alpha}| < |\tau_{\gamma\alpha}|$, which was to be proved.

A consequence of the theorem proved is the possibility of determining the parameter χ for the stationary state resulting from shear in the case where a decrease in shearing stresses leads to equality in the slip conditions (20) and (21).

From the asymmetry of (25) it follows that the strained stress $\sigma^{(1)}_{ij}$ is not coaxial to the total distortion C_{ij} from (2), which was reveled experimentally for anisotropic defect materials or materials with anisotropy induced during deformation.

3. Pore as a Divergence of the Interface. As in Sec. 2, deformation of a small pseudocontinuous neighborhood is considered with allowance for relations (1)–(17). A case is also possible where the normal stresses calculated from (16) are tensile:

$$\sigma_{\alpha\alpha}^{(1)} > 0, \tag{31}$$

i.e., the interface edges diverge with formation of a pore cavity. One can say that the cavity is formed with opening in the α-direction and both sides of the neighborhood diverge, deforming as continuum near the free surface.

Generally, we assume that the tangential stresses calculated from (17) are not equal to zero for the area considered. Then, the slip conditions (20) and (21) with allowance for (31) is simplified:

$$f_\alpha(\sigma_{\alpha\alpha}^{(0)}, \tau_\alpha^{(0)}) \equiv \tau_\alpha^{(0)} \neq 0; \tag{32}$$

the divergence of the edges and formation of the pore occur simultaneously with irreversible slip over this area. Slip parameters are calculated from relations (25)–(27). Description of shear over the opened interface differs from the description of slip over the closed surface only by the condition of shear termination. In the case considered, the shear is terminated when the tangential stress $\tau_\alpha^{(0)}$ on the area decreases to zero [10].

According to [11], for the opened interface, the normal component $\sigma_{\alpha\alpha}^{(0)}$ is equal to zero. In this case, the principal directions of the reversible strain tensor and the stress tensor are established according to the orientation of the opened interface, and the isotropic elastic relations (6) take the form [10, 11]:

$$\sigma_{\alpha\alpha}^{(0)} = \lambda(\varepsilon_{\alpha\alpha}^{\rm con} + \varepsilon_{\beta\beta}^{(0)} + \varepsilon_{\gamma\gamma}^{(0)}) + 2G\varepsilon_{\alpha\alpha}^{\rm con} = 0, \quad \sigma_{\beta\beta}^{(0)} = \lambda(\varepsilon_{\alpha\alpha}^{\rm con} + \varepsilon_{\beta\beta}^{(0)} + \varepsilon_{\gamma\gamma}^{(0)}) + 2G\varepsilon_{\beta\beta}^{(0)},$$
$$\sigma_{\gamma\gamma}^{(0)} = \lambda(\varepsilon_{\alpha\alpha}^{\rm con} + \varepsilon_{\beta\beta}^{(0)} + \varepsilon_{\gamma\gamma}^{(0)}) + 2G\varepsilon_{\gamma\gamma}^{(0)}. \tag{33}$$

System (33) contains the two components $\varepsilon_{\beta\beta}^{(0)}$ and $\varepsilon_{\gamma\gamma}^{(0)}$ known from relation (30) and the three unknown components: the stresses perpendicular to the interface [$\sigma_{\beta\beta}^{(0)}$ and $\sigma_{\gamma\gamma}^{(0)}$] and the strains of the continuous (continual) parts of the region perpendicular to the interface ($\varepsilon_{\alpha\alpha}^{\rm con}$). In this case, the uniformly deformed neighborhood is characterized by one more parameter, in addition to the general strain of the region in the α-direction — $\varepsilon_{\alpha\alpha}^{(0)}$. Thus, the strain of the region as a whole in the α-direction and the strains of its continual parts in the same direction are different as a result of occurrence of the cavity. Since the total strain in the α-direction $\varepsilon_{\alpha\alpha}^{(0)}$, the corresponding distortion $C_{\alpha\alpha}^{(0)}$, and the strain of the continuous parts $\varepsilon_{\alpha\alpha}^{\rm con}$ are considered in relation to the unified unstrained configuration, as the measure of opening of the cavity, we can use the quantity

$$\varepsilon_{\alpha\alpha}^{pr} = \varepsilon_{\alpha\alpha}^{(0)} - \varepsilon_{\alpha\alpha}^{\rm con} = C_{\alpha\alpha}^{(0)} - 1 - \varepsilon_{\alpha\alpha}^{\rm con}, \tag{34}$$

which corresponds to the porosity factor used in classical soil mechanics but also describes the anisotropy of the region considered.

4. Free-Flowing Media as Media with a Continuous Set of Variously Oriented Slip Areas. Free-flowing media are usually treated as media containing a continuous set of interface systems of arbitrary direction. In this case, the most intensely loaded slip surface can be found [12]. The most intensely loaded shear areas are perpendicular to the planes of action of the largest and smallest stresses σ_1–σ_3. The normal ($\sigma_{\alpha\alpha}$) and tangential (τ_α) components on the α-area have the form

$$\sigma_{\alpha\alpha} = \sigma_1 \cos^2 \alpha + \sigma_3 \sin^2 \alpha, \qquad \tau_\alpha = \sin 2\alpha(\sigma_3 - \sigma_1)/2, \tag{35}$$

where α is the slope of the normal to the direction of principal stresses σ_1. The difference of shearing and restraining forces is

$$\Delta\tau = \tau_\alpha + k\sigma_{\alpha\alpha} = \sin 2\alpha(\sigma_3 - \sigma_1)/2 + k(\sigma_1 \cos^2 \alpha + \sigma_3 \sin^2 \alpha). \tag{36}$$

It is maximal for a pair of areas with the slope of the normal lines

$$\alpha = \pm\Big(\frac{\pi}{4} - \frac{1}{2} \arctan k\Big) \tag{37}$$

in the strained state. The orientations of the maximum shear areas in the unstrained state are determined from the inverse relations (10).

5. Local Slips of Free-Flowing Media by Systems of Variously Oriented Areas. A free-flowing medium can be treated as an elastic body containing a set of variously oriented interfaces [10, 13, 14]. Then, the stresses acting on each of the interfaces should be analyzed by relations (10)–(17), choosing surfaces where stresses,

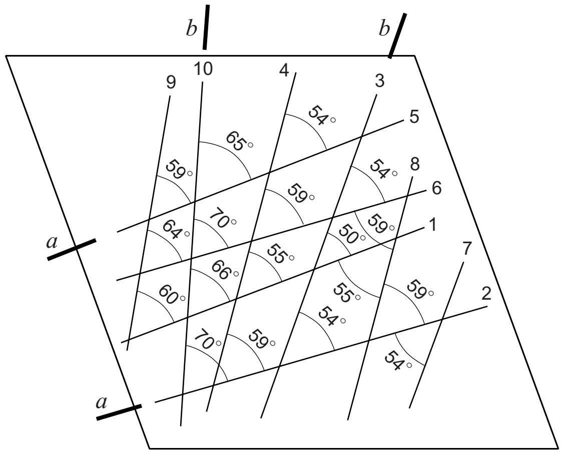

Fig. 1

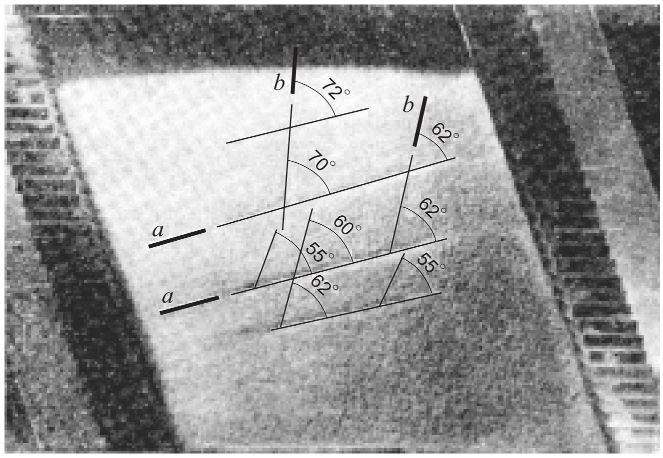

Fig. 2

Fig. 1. Orientation of the calculated sequence of slip areas with sloping of the region at an angle of 20° (first step): 1–10) slip areas of highest intensity in order of occurrence; a and b are the orientations of the areas for the plus (upper sign) and minus (lower sign) in (37), respectively.

Fig. 2. Flat sloping of dry sand in the experiment of [8] (first step): a and b are, respectively, the orientations of areas for the plus (upper sign) and minus (lower sign) in (37).

according to the adopted slip conditions (20) and (21), exceed the limit of the equilibrium state. We assume that the most intensely loaded area is displaced with the highest velocity. At the same time, deformation of the region and loading of the areas proceed over finite time Δt, during which the areas are loaded successively. Therefore, slippage over these areas should be considered successively. This approach simplifies consideration of the mutual effect of slips because slip over one area, according to relations (28)–(30), changes the strained stress in the tested neighborhood (and, hence, the stresses over each area) and the order of “overstrained” areas according to the adopted slip conditions (20) and (21), and makes it possible to distinguish a new most intensely loaded area. In this case, one does not need to introduce special functions reflecting the mutual effect of various slip areas [7]. It should be noted that in free-flowing media there is no “blocking” [4] of some slip areas due to shears over other areas. Thus, complete irreversible slip transformation occurs by successive elementary slips over α-interfaces ($\alpha = 1, 2, \ldots, l$):

$$\boldsymbol{A} = \boldsymbol{B}_{(l)}\boldsymbol{C}^{(0)}_{(l)}\boldsymbol{A}^{\rm sh}_{(l)}\boldsymbol{A}^{\rm sh}_{(l-1)}, \ldots, \boldsymbol{A}^{\rm sh}_{(2)}\boldsymbol{A}^{\rm sh}_{(1)}. \tag{38}$$

As in the case with simple slip over the interface (30), in the last relation, one can distinguish the strain gradient relative to the new unstrained configuration attained after the lth step of slip:

$$A^0_{(l)} = B_{(l)}C^{(0)}_{(l)}. \tag{39}$$

Thus, the facts of successive slips are accumulated in a single current unstrained configuration, which is sufficient to store. Thus, the entire history of irreversible shear deformation is stored in the unstrained configuration of the region. However, the slips change the orientations of all interfaces (10)–(13), which should be appropriately corrected after each distortion.

Since slippage continuously changes the orientations of all areas, it is expedient to determine the slip parameter from the time dependence (27), approaching the actual strain trajectory by small steps, to determine the magnitude of the time step from the convergence condition.

Results of numerical simulation of the localization of slip areas (Fig. 1) for a free-flowing medium were compared to the corresponding results for flat sloping in dry sand [8] (Fig. 2). The slip areas are numbered by the author in order of occurrence. The dry friction angle over slip areas is set equal to 31° (natural sloping of sand) [8].

In the numerical experiment, as a first step, we specified hard loading by the strain gradient (1) corresponding to a slope of 20° (in the experiment of [8], the sloping frame was rotated counter-clockwise through an angle of 20°). The resulting stress was calculated from relations (6), the position of slip areas was obtained from relation (37), and the normal and tangential stresses acting on the areas were determined from relations (35). The overstress level calculated from (36) was unloaded by local shears [see (25)–(27)] shown in Fig. 1. A feature of this shear

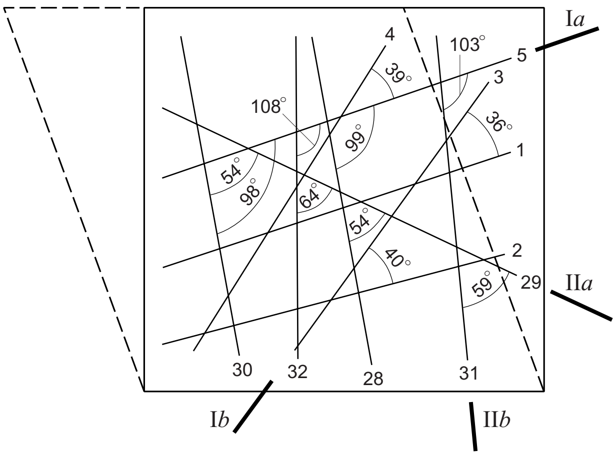

Fig. 3. Orientation of the calculated sequence of slip areas after restoration of the initial configuration of region (second step): the dashed lines show the previous configuration; I and II are, respectively, the slip areas at the first and second steps, respectively; a and b are the orientations of areas for the plus (upper sign) and minus (lower sign) in (37).

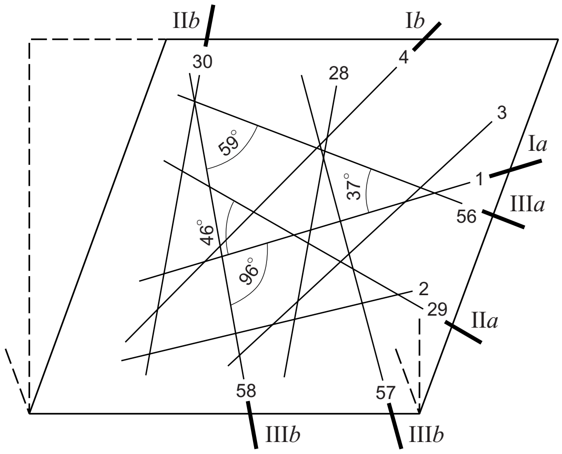

Fig. 4

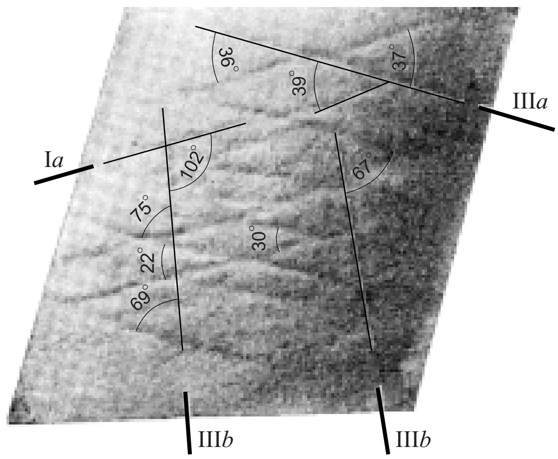

Fig. 5

Fig. 4. Orientation of the calculated sequence of slip areas in reverse sloping of the region through an angle of 20° (third step): the dashed lines show the previous configurations; I, II, and III are the slip areas at the first, second, and third stages, respectively; a and b are, respectively, the orientations of areas for the plus (upper sign) and minus (lower sign) in (37).

Fig. 5. Slip lines for reverse sloping (third step) of dry sand in the experiment of [8].

loading is that it leads to tensions and opening of pores, according to relations (31)–(34), in the direction of the elongating diagonal. The pores are rapidly filled during the slips described by (20)–(30) and (37)–(39). Therefore, the simulated quasistationary loading is a sequence of these two nonequilibrium processes. The choice between the two equally loaded shear areas (37) was implemented in a random fashion. The distribution of the areas over the region was also assumed to be random. For visualization of the numerical experiment, the position of ten slips of highest intensity (26) was traced. The numbering of slip areas given Fig. 1 corresponds to the order of occurrence. Results of the experiments of [8] with a real free-flowing material (see Fig. 2) showed that the angles between the areas in the experiment are different for different areas. The same result was obtained numerically (see Fig. 1) because the calculations took into account the effect of previous shears on the stress–strained state at the beginning of subsequent shears and on the change of the orientation of previous slip areas. The calculated angles between slip areas (sf. Figs 1 and 2) are also in good agreement with experimental data. In this case, the disalignment of the general strains reached 4.6°. As a result of the slips, the stress relaxes to zero and the entire strain "imparted" to the region becomes irreversible.

At the second step of the numerical experiment, hard loading was specified by reverse sloping through an angle of 20° and restoration of the initial configuration of the region (the sloping frame in the experiment of [8] was rotated clockwise through an angle of 20°, and the angle of total sloping is equal to zero). The resulting stresses were decreased by slips of the new set (Fig. 3, areas IIa and IIb) up to complete disappearance. In this case, the previous shear areas changed orientation. For visualization, we chose 10 areas with largest magnitudes of slips (26) over the entire history of loading: area Nos. 1–5 and 28–32 (see Fig. 3).

At the third step of the numerical experiment, hard loading was specified by subsequent reverse sloping through an angle of 20° (the sloping frame in the experiment of [8] was rotated clockwise through an angle of 20°, so that the total angle of rotation was 20° in this direction). The resulting strained state is changed by slips of a new, third, set (Fig. 4, areas IIIa and IIIb). In this case, the orientations of the previous shear areas change again. For visualization, we chose 10 areas of largest slips: area Nos. 1–4, 28–30, 56, 57, and 59. The results of the numerical experiment after the third loading step are in qualitative agreement to the pattern of slips for a real free-flowing material [8] given in Fig. 5, where one can clearly see slips at various steps of sloping. From the calculation results (see Figs. 1, 3, and 4) it follows that despite the extreme simplicity of the slip condition (37), the slip areas are self-organized in systems whose orientation show certain regularities. The fact that this effect is due to allowance for the effect of previous slips on subsequent slips is very important and characterizes the observed effect of self-organization of slips as occurrence of "the complex" (see Figs. 1, 3, and 4) from "the simple" (37). The calculated angles of relative orientation of areas are close to the experimental values (sf. Figs. 2 and 5).

The author thank V. A. Babeshko, E. V. Glushkov, and N. V. Glushkova for assistance and L. M. Zubov for detailed discussions of the calculation procedure.

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